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THE EFFECT OF TURBINE BLADE COOLING  
ON EFFICIENCY OF A SIMPLE  
GAS TURBINE POWER PLANT

BY

W. M. ROHSENOW

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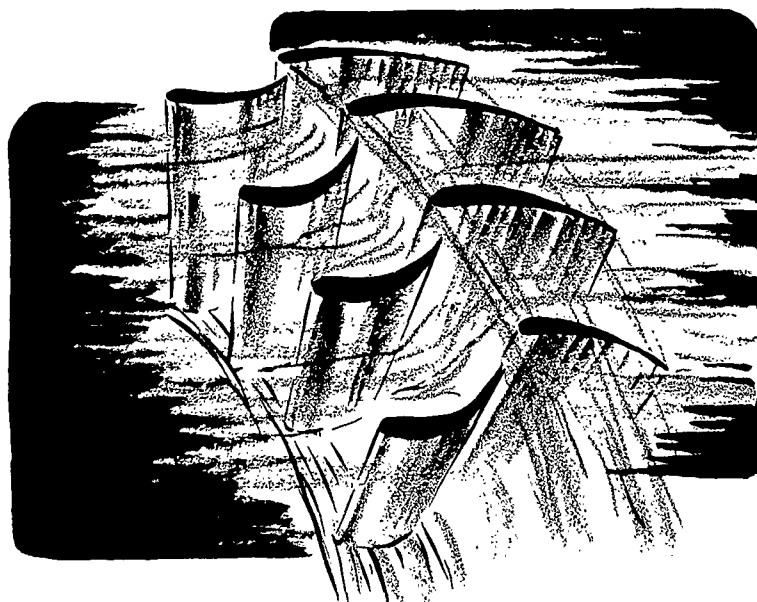
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EFFECT OF TURBINE BLADE COOLING ON EFFICIENCY  
OF A SIMPLE GAS TURBINE POWER PLANT

by

W. M. Rohsenow\*

The efficiency of a simple gas turbine cycle can be increased by designing it to operate at higher turbine inlet temperatures: however, the extent to which this temperature can be increased depends upon the ability of the metal of the turbine to withstand the high temperatures without deterioration. Uncooled turbines have been run for reasonable lengths of time at inlet temperatures of 1500 F, but such turbines must employ expensive and often commercially unavailable materials. The less expensive and more available materials are limited to operation with inlet gas temperatures of around 1000 F.

In order to operate these turbines at higher temperatures it is necessary to cool the blades and possibly the casing and rotors in the earlier stages until the gases are reduced to a value between 1000 F and 1500 F depending on the kind of metal employed. Various methods of cooling turbine blades have been investigated by the NACA and published in various unclassified technical reports since 1947. A survey of some of this work is presented by Ellerbrock (1).

The present work is an investigation of the effects of blade cooling on the thermodynamic performance of a simple gas turbine plant shown in figure 1. This study is of the nature of an exploratory study to show general trends. Because the analysis is not limited to a particular design of gas turbine its results are in a sense qualitative

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\* Associate Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.

but show definite trends. Three design situations are investigated. With currently available metal alloys gas turbines can be expected to be operated uncooled at temperatures around 1500 F. With ordinary steels requiring only readily available material they can be operated at temperatures around 1000 F. In the region where the gases exceed these temperatures the turbine must be cooled. Hence in this analysis the turbine is considered to have two parts - A, a cooled part, and B, an uncooled part, figure 1. The turbine is cooled until the temperature  $T_1$  is reached along its condition curve. The magnitude of  $T_1$  depends upon the kind of metal used in the turbine construction.

The following three conditions were investigated for the effect on thermal efficiency and specific air consumption for turbines with an infinite number of stages:

- 1)  $T_1 = 1500$  F,  $T_3$  varying
- 2)  $T_1 = 1000$  F,  $T_3$  varying
- 3)  $T_1 =$  varying,  $T_3 = 1500$  F.

Also the effect of finite number of stages was investigated for condition (1) and (2) above.

#### Method of Analysis

For this analysis the compressor efficiency was assumed to be 85%, and the turbine polytropic efficiency (efficiency of an infinitesimal stage) was assumed to be 88% for both the cooled and uncooled portions of the turbine and for a turbine with a finite number of stages the stage efficiency was assumed to be 85%. Hawthorne (3) showed that the difference in  $\eta_p$  for a cooled and an uncooled stage was probably of the order of 1 to 2%, changing slightly with Mach number and type of stage. The actual turbine efficiency for the uncooled part of the turbine was calculated from

$$\eta_t = R \eta_p \quad (1)$$

where the reheat factor (2) is given by

$$\eta_{co} = \frac{1}{\eta_p} \left( \frac{1 - r_B \eta_p^{\frac{k-1}{k}}}{1 - r_B^{\frac{k-1}{k}}} \right) \quad (2)$$

For the cooled portion of the turbine Appendix B shows that equations (1) and (2) would apply if  $\eta_p$  and  $r_B$  in these equations are replaced by  $\eta_q$  and  $r_A$  where

$$\eta_q = \eta_p \left( 1 + \frac{Q}{W_A} \right) \quad (3)$$

assuming  $Q/W_A$  is uniform along the length of the cooled portion.

Sample results of calculation employing these equations along with the Gas Tables (5) are shown in figure (2) for  $Q/W_A = 0.1$  and  $0.3$  in the cooled part of the turbine. In each of these curves the turbine was cooled to  $T_1 = 1500$  F. Of course, when  $T_3 = 1500$  F there is no cooling in the turbine. Curves similar to these were calculated and drawn to investigate the various conditions listed in the preceding section. A sample calculation is shown in Appendix A.

In order to interpret the results of these calculations it is necessary to investigate how  $Q/W_A$  changes as the design value of  $T_3$  changes. The required amount of cooling, hence  $Q/W_A$ , increases as  $T_3$  is assumed higher because the rate of heat transfer increases with the difference between gas and metal temperature. The following approximate analysis is made to aid in this interpretation.

The maximum work per stage of infinite staged turbine or best efficiency of a single stage turbine is obtained when the magnitude of the term

$$\frac{W}{u^2/2g_o J} = K_1 \quad (4)$$

is approximately  $K_1 = 4$  for impulse blading and  $K_1 = 2$  for reaction blading (ref. 6). Actual values will be slightly less than these.

An approximate relation for the heat transfer per lb. of fluid flowing in a cascade of blades is shown in Appendix C to be

$$Q = c_p (T_o - T_w) (e^{f_o/2} - 1) \quad (5)$$

or approximately twice this value for a turbine stage. This relation assumes the Reynolds analogy between friction and heat transfer to exist.

Then dividing eq. (5) by eq. (4) an expression for  $Q/W$  for a stage can be written as

$$\frac{Q}{W} = \frac{2 c_p (T_o - T_w) (e^{f_o/2} - 1)}{K_1 (u^2/2g_oJ)} \quad (6)$$

If all stages of a multistage turbine have the same velocity diagram the enthalpy drop (and hence temperature drop) per stage of an uncooled turbine is uniform throughout the turbine. Then the temperature varies nearly linearly with axial distance for a uniform blade width for all stages. This is also approximately true for a cooled turbine. If the blade temperature in the first part of the turbine is to be maintained at  $T_w = 1500$  F to a point where the gas temperature is  $T_1 = 1500$  F, then the heat transfer rate will vary linearly with axial position for this part of the turbine. To obtain an average value of  $Q/W_A$ , the temperature  $T_o$  may be interpreted as the mean value between  $T_3$  and  $T_w$  (or  $T_1$ ). Then

$$T_o = \frac{1}{2}(T_3 + T_1) \quad (7)$$

Consider an impulse stage with  $\rho = 0.08$  and  $u = 1000$  ft/sec., then  $(Q/W_A) (T_o - T_w) = 0.000275$  from eq. (6) with  $C_p = 0.27$  Btu/lb F. For  $Q/W_A = 0.1$ ,  $(T_o - T_w) = 363.5$  F; then from equation (7) with  $T_w = 1500$  F,  $T_3 = 2227$  F, point M figure 3. A series of such points is plotted on figure 4 resulting in the solid lines representing the attainable efficiency with blade cooling for an impulse stage with  $\rho_o = 0.08$  and  $u = 1000$  ft/sec.

Continuing this type of calculation for various values of  $u$  and  $\rho_o$  for both reaction ( $K_1 = 4$ ) and impulse ( $K_1 = 2$ ) stages results in the curves of figures 3 through 12. The effect of cooling on cycle efficiency and specific air consumption is shown for maximum efficiency points and also for points at  $r = 15$ .

The preceding results were for a turbine having an infinite number of stages. In actuality these results have neglected the effect of the leaving loss normally associated with turbines having a finite number of stages. To investigate the effect of the leaving loss associated with turbines having finite size stages the calculation proceeds in the same manner as before except that the reheat factor for the uncooled turbine is calculated from (ref.2)

$$\mathcal{R}_N = \frac{\left[1 - \tau_B^{\frac{1}{N} \frac{k-1}{k}}\right] \left[1 - \tau_B^{\eta_p \frac{k-1}{k}}\right]}{\left[1 - \tau_B^{\frac{\eta_p \frac{k-1}{k}}{N} \frac{k-1}{k}}\right] \left[1 - \tau_B^{\frac{k-1}{k}}\right]} \quad \text{--- (8)}$$

For a cooled turbine an analysis similar to that in appendix B will show that equation (8) is valid if  $\eta_p$  is replaced by  $\eta_q$  and  $\tau_B$  by  $\tau_A$ .

In the finite staged turbine calculations the stage efficiency  $\eta_{St}$  was selected as 85%. Then to calculate  $\eta_N$  above  $\eta_p$  and  $\eta_q$  are found from the following relations (Appendix D)

$$\text{(Uncooled)} \quad \eta_{St} = \frac{1 - \left(\frac{\eta_p}{N}\right)^{\frac{k-1}{k}}}{1 - \left(\frac{1}{N}\right)^{\frac{k-1}{k}}} \quad \text{--- (9)}$$

$$\text{(Cooled)} \quad \eta_{St} = \frac{1}{1 + Q/W} \cdot \frac{1 - \left(\frac{\eta_q}{N}\right)^{\frac{k-1}{k}}}{1 - \left(\frac{1}{N}\right)^{\frac{k-1}{k}}} \quad \text{--- (10)}$$

Also for a turbine with a finite number of stages there will be a "leaving loss" associated with the velocity of the gas leaving the last stage. If the blade speed is selected to cause this leaving velocity to be in the axial direction and if the axial velocity is assumed to be uniform throughout the turbine the kinetic energy of the leaving gas for impulse stages will be approximately

$$\frac{V_{axial}^2}{2 g_o J} = \frac{\Delta h_{St} \sin^2 \beta}{N} \quad \text{--- (9)}$$

since the nozzle velocity neglecting frictional effects is given by  $\sqrt{2 g_o J \Delta h_{ss}}$  and  $\Delta h_{ss} = \Delta h_{St} / N$  and  $V_{axial} = V_n \sin \beta$ . This kinetic energy must be subtracted from the turbine work calculated by neglecting the leaving loss in order to obtain the turbine with a finite number of stages.

The results are shown in figures 13 and 14 showing the effects on cycle efficiency of magnitudes of  $T_3$ ,  $T_1$ ,  $N$  and  $\xi_o$ . For each point on these curves the blade speed was taken as

$$U = \frac{1}{2} \cos \beta \sqrt{2 g_o J \Delta h_{St} / N} \quad \text{--- (10)}$$

which is the approximate relation required to have an axial leaving velocity.



### Discussion of Results

The values of efficiency plotted in figures 3 through 6 are the maximum efficiency points of curves similar to those of figure 2. The dashed lines represent constant  $Q/W_A T_3$  values. From equations (6) and (7) values of  $Q/W_A$  associated with a value of  $T_3$  may be obtained. Then the heavier solid curves of figures 3 through 6 may be drawn showing the effects of  $\xi_o$  and  $u$ .

A reasonable design value for  $u$  is around 1000 ft/sec and a reasonable value for  $\xi_o$  to obtain  $\eta_p = 0.88$  is probably in the range 0.05 - 0.08. The effect of changes in the magnitude of profile loss factor  $\xi_o$  on the value of  $\eta_p$  is not great since the profile loss is only 1/3 to 1/2 of the stage loss factor. From figures 3 and 5 it appears that with  $T_1 = 1500$  F it is unprofitable to raise the design inlet temperature of a cooled reaction turbine beyond 2000 - 2500 F and of a cooled impulse turbine beyond 3000 F. Here it was assumed that only the blades (rotor and stator) were cooled. If any part of the casing is also cooled or if the value of  $\xi_o$  is higher than 0.05 - 0.08 the peak of these efficiency curves would be at lower values of  $T_3$ .

The cycle efficiency employing the uncooled turbine with  $T_3 = 1500$  F is about 35%. With cooling it appears possible to attain efficiencies in range 40 - 45% by raising  $T_3$  to values in the range 2500 - 3000 F. Simultaneously the specific air consumption decreases as shown in figure 8 and the pressure ratio at maximum cycle efficiency rises from around 15 to around 50 or 60. This reduction in

specific air consumption results in smaller turbine and compressor diameters, but the larger pressure ratios require more stages and thicker casings and piping. Because of this it might be desirable to avoid the use of high pressure ratios; so curves for cycle efficiency and air consumption were prepared for a pressure ratio of 15 in figures 7 and 8. Under these conditions it appears undesirable to use  $T_3$  beyond about 2000 F with an unimpressible rise in efficiency from 35% only to about 38 - 39%.

Figures 9 and 10 are drawn for  $T_1 = 1000$  F and are similar to figures 3 and 7 which were drawn for  $T_1 = 1500$  F. These curves are applicable to a cooled turbine constructed of less expensive and more readily available material. Of course the cycle with an uncooled turbine at  $T_3 = 1000$  F has much smaller efficiency ( $\eta = 23\%$ ) than with  $T_3 = 1500$  F ( $\eta = 35\%$ ). However, when a cooled turbine is employed with  $T_1 = 1000$  F and  $T_3$  raised to 2500 - 3000 F the cycle efficiency is again in the range of 40 - 45%, not very much less than the cycle with a cooled turbine with  $T_1 = 1500$  F. Similarly if  $r = 15$  and  $T_3$  is allowed to take on a value of 2000 F an efficiency of about 36% is obtained which is only a few per cent less than attainable values at  $r = 15$  and  $T_3 = 2000$  F with  $T_1 = 1500$  F. Figures 11 and 12 were drawn to show the effect of  $T_1$  on cycle efficiency with  $T_3 = 1500$  F.

The effect of the leaving loss associated with a finite staged turbine is shown in figure 13. The upper curve of each set is for the uncooled turbine and shows a decreasing efficiency as number of stages is decreased. This is due to the increased "leaving loss" as number of stages is decreased. For the cooled turbine the efficiency

passes through a maximum value for some finite number of stages because as the number of stages increases the effect of the heat loss becomes greater and offsets the effect of the decreasing leaving loss. Furthermore the point of maximum efficiency occurs at a smaller number of stages as the turbine inlet temperature is raised since the heat transferred increases with  $T_3$  for a fixed  $T_1$ . From the curves of figure 13 it is observed that the cycle efficiency change is less than 1% for cooled turbines having numbers of stages in the range of 2 to 10 for the cycle conditions investigated.

Figure 14 shows that the effect of different values of loss factor  $\xi_0$  on the cycle efficiency. It is observed that the cycle efficiency curves level-off as turbine inlet temperature increases.

In this analysis it has been assumed the blade temperatures selected could be obtained. Inherent in any liquid or gaseous coolant system for cooling the blades there would be an energy loss associated with circulating and cooling the coolant. The effect of this would be to flatten the efficiency vs inlet temperature curves more than those shown in the present results. The actual energy loss associated with the coolant system depends on the particular design of this system. The absolute magnitude of cycle efficiency will depend on the detailed design of the coolant system; nevertheless the results of this analysis are valid for comparison purposes.

### Conclusion

The results of this analysis suggest that the real value of blade cooling is associated with turbines constructed of less expensive and more readily available materials. Turbines with design inlet temperatures of 2000 - 2500 F requiring cooling to 1000 F can obtain cycle efficiencies within 1 or 2% of those obtained by turbines requiring cooling to 1500 F but designed with inlet temperature in the same range of 2000 - 2500 F.

### Acknowledgment

This program of investigation was originally undertaken by Dr. W. R. Hawthorne, now at Cambridge University, Cambridge, England. The present report represents an application of the methods developed by him. Calculations were performed by Mrs. Antonia B. Walker and Mr. T. E. Luzzi.

NOMENCLATURE

G	Rate of flow per unit cross-sectional area, lb/hr.ft <sup>3</sup>
J	778 ft.lb/Btu.
K	Defined in equation (4).
N <sup>1</sup>	Number of stages in a turbine.
P	Wetted-perimeter for flow through the blade passage.
Q/W <sub>A</sub>	Heat transfer to cooled blades per unit turbine work.
Q	Heat transferred to cooled blade per lb. of fluid.
S	Cross-sectional area for flow through blade passage.
T <sub>1</sub>	Exit temperature of turbine part A.
T <sub>0</sub>	Stagnation temperature.
T <sub>w</sub>	Wall temperature taken equal to T <sub>1</sub> ).
V <sub>axial</sub>	Axial velocity leaving last stage
V <sub>n</sub>	Velocity of jet leaving turbine stage nozzle blades.
W <sub>A</sub>	Work per lb. of fluid done by cooled part of turbine.
W <sub>B</sub>	Work per lb. of fluid done by uncooled part of turbine.
W <sub>C</sub>	Compressor work per lb. of fluid.
W <sub>T</sub>	Turbine work per lb. of fluid.
(W <sub>T</sub> -W <sub>C</sub> )	Cycle net work per lb. of fluid.
c <sub>p</sub>	Specific heat at constant pressure, Btu/lb F.
g <sub>0</sub>	32.2 lbmft/lbf sec <sup>2</sup> , conversion factor.
h	enthalpy, Btu/lb.
Δ h <sup>stg</sup>	Enthalpy drop across a stage.
Δ h <sub>st</sub>	Isentropic enthalpy difference across a turbine
h <sub>ss</sub>	Isentropic enthalpy difference across a stage.
r	Pressure ratio, less than unity
r <sub>A</sub>	Pressure ratio across cooled part of turbine, p <sub>1</sub> /p <sub>3</sub>
r <sub>B</sub>	Pressure ratio across uncooled part of turbine, p <sub>4</sub> /p <sub>1</sub>
u	Blade speed, ft/sec.
α	Surface coefficient of heat transfer, Btu/hr ft <sup>2</sup> F.
λ	Loss factor defined by equation (C-4) of Appendix.
μ	Nozzle angle.
ξ <sub>p</sub>	Profile loss factor.
η	Cycle efficiency.
η <sub>c</sub>	Compressor efficiency.
η <sub>stg</sub>	Stage efficiency of a finite turbine stage.
η <sub>t</sub>	Turbine efficiency
η <sub>p</sub>	Polytropic turbine efficiency (efficiency of an infinitesimal stage)
η <sub>q</sub> = η <sub>p</sub> (1 + Q/W <sub>A</sub> )	for a cooled part of turbine (reference 4).
p <sub>r</sub>	Relative pressure as used in reference 5.
R <sub>∞</sub>	Reheat factor.

APPENDIX ASample Calculation

- (a) Compressor Work:  $T_1 = 520^\circ \text{ R}$ ,  $\eta_c = 0.85$ ,  $r = 10$

From Gas Tables: (5)  $h_1 = 124.27$ ,  $p_{r1} = 1.2147$

then  $p_{r2} = 12.147$  and  $h_{2s} = 240.14$

So  $W_c = (h_{2s} - h_1) \eta_c = 136.32$ .

- (b) Turbine Work:  $T_3 = 2460^\circ \text{ R}$ ,  $\eta_p = 0.88$ ,  $r = 10$ ,  $Q/W_A = 0.2$ ,  $T_i = 1960^\circ \text{ R}$

(1) Cooled Part

$$W_A = h_3 - h_1 - Q = \frac{h_3 - h_1}{1 + Q/W_A}$$

From the Gas Tables:  $h_3 = 634.34$ ,  $h_1 = 493.64$ ,  $p_{r3} = 407.3$

$$W_A = \frac{634.34 - 493.64}{1.2} = 117.25$$

Now  $h_{is}$  has to be found by trial and error:

$$W_A = \eta_p \phi_A (h_3 - h_{is})$$

Assume  $\phi_A = 1$  then approximately  $h_{is} = h_3 - \frac{W_A}{\eta_p}$

$$h_{is} = 634.34 - \frac{117.25}{.88} = 501.10$$

Gas tables give approximate  $p_{ris} = 169.5$

$$p_{r3}/p_{ris} = 2.40$$

From equations (2) and (3):  $\eta_q = \eta_p (1 + Q/W_A) = 0.88(1.2) = 1.056$

This value and  $r = 2.40$  give  $\phi_A = .9927$

This gives a new value of  $h_{is}$ , namely

$$h_{is} = h_3 - \frac{W_A}{\eta_p \phi_A} = 634.34 - \frac{117.25}{(.88)(.9927)} = 500.13$$

Repeating the above procedure gives  $p_{r3}/p_{ris} = 2.42 = r_A$  and the same value of  $\phi_A$  as above.

(2) Uncooled part,  $W_B$

$$p_{r1}/p_{r4} = r/r_A = 10/2.42 = 4.13$$

For  $\eta_q = \eta_p = .88$  and pressure ratio = 4.13

$$\phi_B = 1.0223$$

$$p_{r4s} = \frac{p_{r1}}{p_{r1}/p_{r4}} = \frac{160.37}{4.13} = 38.83$$

From Gas Tables  $h_{4s} = 333.45$

APPENDIX A (Continuation)

$$W_B = \eta_p Q_B (h_1 - h_{4s}) = (.88)(1.0223)(493.64 - 333.45) = 144.12$$

$$W_T = W_A + W_B = 261.37$$

(c) Heat added  $Q_{in} = h_3 - h_1 - W_C = 373.75$

(d) Cycle Efficiency =  $\frac{W_T - W_C}{Q_{in}} = 33.5\%$

## APPENDIX B

### Derivation of Equations (2) & (3) for a Cooled Turbine of Infinite Stages.

A stage efficiency for the cooled part of the turbine with an infinite number of stages may be defined as (reference 4)

$$\eta_q \equiv \frac{W + Q}{\Delta h_{ss}} \quad \text{--- (B-1)}$$

neglecting kinetic changes across the stage. Also for the cooled turbine a polytropic efficiency may be defined as

$$\eta_p \equiv \frac{W}{\Delta h_{ss}} \quad \text{--- (B-2)}$$

Then dividing one of these by the other

$$\eta_q = \eta_p \left(1 + \frac{Q}{W}\right) \quad \text{--- (B-3)}$$

which is equation (3).

For an infinitesimal stage

$$dh = (\delta W + \delta Q) = \eta_q dh_{ss} = \eta_q v dp \quad \text{--- (B-4)}$$

since for an isentropic process  $dh_{ss} = v dp$

Substituting  $C_p dT$  for  $dh$  and  $RT/p$  for  $v$  in equation (B-4) and integrating between any two points, m and n, along the condition curve of the turbine

$$\frac{T_m}{T_n} = \left(\frac{p_m}{p_n}\right)^{\eta_q \frac{(k-1)}{k}} \quad \text{--- (B-5)}$$

The reheat factor for a cooled turbine with an infinite number of stages is defined as

$$R_{re} \equiv \frac{\sum \Delta h_{ss}}{\Delta h_{st}} = \frac{(1/\eta_q) C_p (T_3 - T_1)}{C_p T_3 \left[1 - r^{\frac{k-1}{k}}\right]} \quad \text{--- (B-6)}$$

Then substituting equation (B-5) into (B-6)

$$R_{re} = \frac{1}{\eta_q} \frac{1 - (r_A)^{\eta_q \frac{k-1}{k}}}{1 - (r_A)^{\frac{k-1}{k}}} \quad \text{--- (B-7)}$$

which is the same form as eq. (2) with  $\eta_p$  replaced by  $\eta_q$  and  $r_B$  by  $r_A$ .



# APPENDIX C

## Derivation of Equation (5)

Hawthorne (3) suggests the following approximate analysis for determining the heat transfer per lb. of fluid flowing across a turbine blade row.

The Reynolds analogy relating momentum transfer and heat transfer is

$$\frac{\alpha}{C_p G} = \frac{f}{2} \quad \text{--- (C-1)}$$

An energy balance for an element of length  $dx$  along the blade surface in the direction of flow is

$$dq = \alpha C_p (T_w - T_o) dx = S G C_p dT_o \quad \text{--- (C-2)}$$

Substituting eq(C-1) into eq(C-2) and integrating between points 1 and 2 at inlet and outlet of the blade passage results in

$$\frac{T_{o1} - T_{o2}}{T_{o2}} = \frac{T_{o2} - T_w}{T_{o2}} \left( e^{\int_0^L \frac{fP}{2S} dx} - 1 \right) \quad \text{--- (C-3)}$$

Defining a loss factor  $\xi_o$  by

$$\xi_o \equiv \int_0^L f \frac{P}{S} dx \quad \text{--- (C-4)}$$

Then since the heat transferred per lb. of fluid is

$$Q = C_p (T_{o1} - T_{o2}) \quad \text{--- (C-5)}$$

eq(C-3) with equations (C-4) and (C-5) become

$$Q = C_p (T_{o2} - T_w) (e^{\xi_o/2} - 1) \quad \text{--- (C-6)}$$

which is eq. (5).

For low Mach number and for the case in which  $(P/S)$  does not vary along the flow path, Hawthorne (3) shows that  $\xi_o = \xi_p$  the profile loss factor. This would correspond to the case of impulse blading. For reaction blading and nozzles having decreasing area in the direction of gas flow and for values of Mach number greater than zero  $\xi_p$  becomes less than  $\xi_o$ . For an area ratio of 1/3 and a Mach number of 0.5 and  $\xi_o = 0.08$ ,  $\xi_p/\xi_o = 0.34$  (ref.3).

APPENDIX C (Continuation)

It is probable that the profile loss factor for reaction blades is smaller than that for impulse blades; hence,  $\xi_o$  may not be much different for the two types of blading. More experimental information is needed to determine values of  $\xi_o$  to be used in this analysis. In the absence of more precise information  $\xi_o$  may be assumed to be the same for both types of blading.

## APPENDIX D

### Derivation of Equations (9) and (10)

The definition of stage efficiency is

$$\eta_{stg} = \frac{W_{stg}}{\Delta h_{ss}} = \frac{\Delta h_{stg} - Q}{\Delta h_{ss}} \text{ --- (D-1)}$$

if the difference between the kinetic energy of the gases entering and leaving the stage is negligible. Then

$$\eta_{stg} = (1 + \frac{Q}{W}) = \frac{\Delta h_{stg}}{\Delta h_{ss}} = \frac{cp(\Delta T_{stg})}{cp(\Delta T_{ss})} \text{ --- (D-2)}$$

Assuming equal pressure ratio per stage and the condition curve equation (B-5) defining  $\eta_q$ , equation (D-2) becomes equation (10). Then if  $Q/W$  is zero (uncooled stage)  $\eta_q = \eta_p$  from equation (B-3) and equation (10) reduces to equation (9).

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# LIST OF ILLUSTRATIONS

- Figure 1 Diagram and Enthalpy - Entropy chart of simple gas turbine process with part A of the turbine cooled.
- Figure 2 Cycle efficiency - pressure ratio plot for  $Q/W_A = 0.2$  and  $0.3$  with  $T_1 = 1500$  F.
- Figure 3 Maximum cycle efficiency versus turbine inlet temperature at various values of  $\beta_0$ . Impulse stages with  $u = 1000$  ft/sec and  $T_1 = 1500$  F.
- Figure 4 Maximum cycle efficiency versus turbine inlet temperature at various values of  $u$ . Impulse stages with  $\beta_0 = 0.05$  and  $T_1 = 1500$  F.
- Figure 5 Maximum cycle efficiency versus turbine inlet temperature at various values of  $\beta_0$ . Reaction stages with  $u = 1000$  Ft/sec and  $T_1 = 1500$  F.
- Figure 6 Maximum cycle efficiency versus turbine inlet temperature at various values of  $u$ . Reaction stages with  $\beta_0 = 0.05$  and  $T_1 = 1500$  F.
- Figure 7 Cycle efficiency versus  $T_3$  at various values of  $\beta_0$  when  $r = 15$ . Impulse stages with  $u = 1000$  ft/sec and  $T_1 = 1500$  F.
- Figure 8 Specific air consumption versus  $T_3$ . Impulse stages with  $u = 1000$  ft/sec,  $\beta_0 = 0.05 - 0.08$  and  $T_1 = 1500$  F.
- Figure 9 Maximum cycle efficiency versus  $T_3$  at various values of  $\beta_0$ . Impulse stages with  $u = 1000$  ft/sec and  $T_1 = 1000$  F.
- Figure 10 Cycle efficiency at  $r = 15$  versus  $T_3$  at various values of  $\beta_0$ . Impulse stages with  $u = 1000$  ft/sec and  $T_1 = 1000$  F.
- Figure 11 Maximum cycle efficiency versus  $T_1$  at various values of  $\beta_0$ . Impulse stages with  $u = 1000$  ft/sec and  $T_3 = 1500$  F.
- Figure 12 Cycle efficiency at  $r = 15$  versus  $T_1$  at various values of  $\beta_0$ . Impulse stages with  $u = 1000$  ft/sec and  $T_3 = 1500$  F.
- Figure 13 Effect of Finite Number of Stages on Cycle Efficiency for Impulse Stages with  $\beta_0 = 0.08$
- Figure 14 Effect of  $T_3$  and  $\beta_0$  on Efficiency of a Cycle with a Turbine of Three Impulse Stages and Pressure Ratio of 4.4.

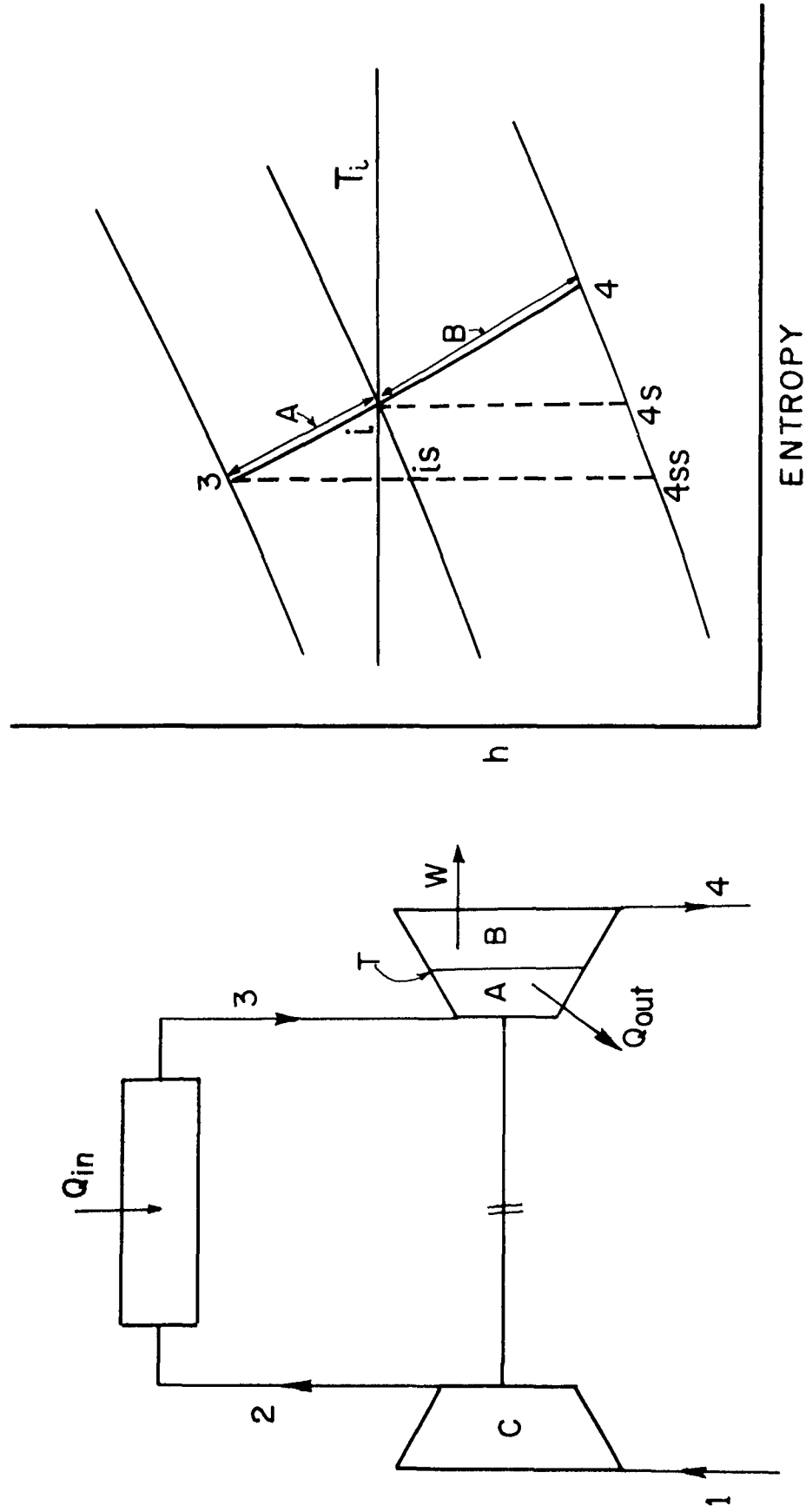


FIG. 1. DIAGRAM AND ENTHALPY-ENTROPY CHART OF SIMPLE GAS TURBINE PROCESS WITH PART A OF THE TURBINE COOLED.

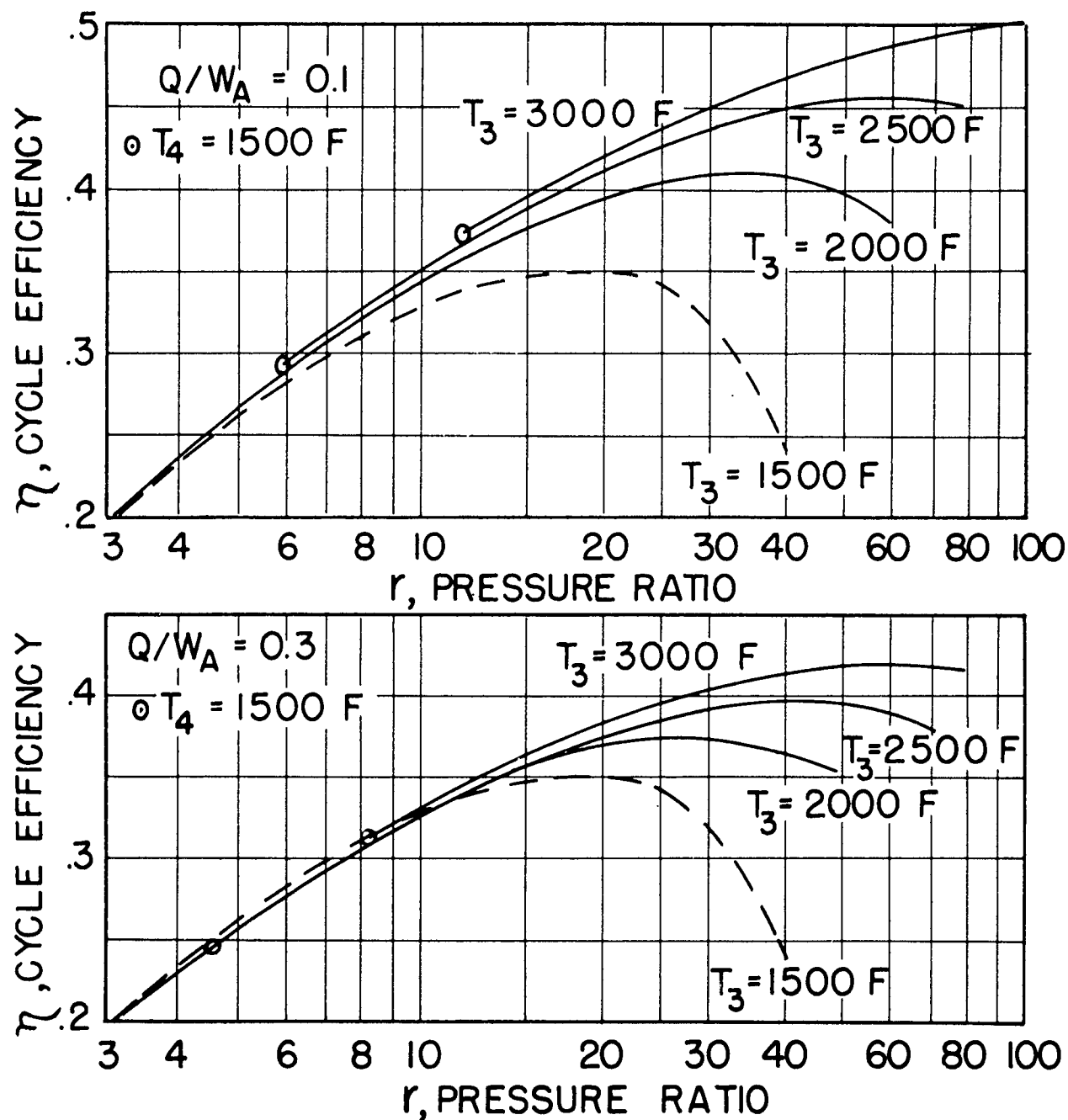


FIG. 2. CYCLE EFFICIENCY — PRESSURE RATIO  
PLOT FOR  $Q/W_A = 0.1$  AND  $0.3$  WITH  $T_i = 1500$  F.

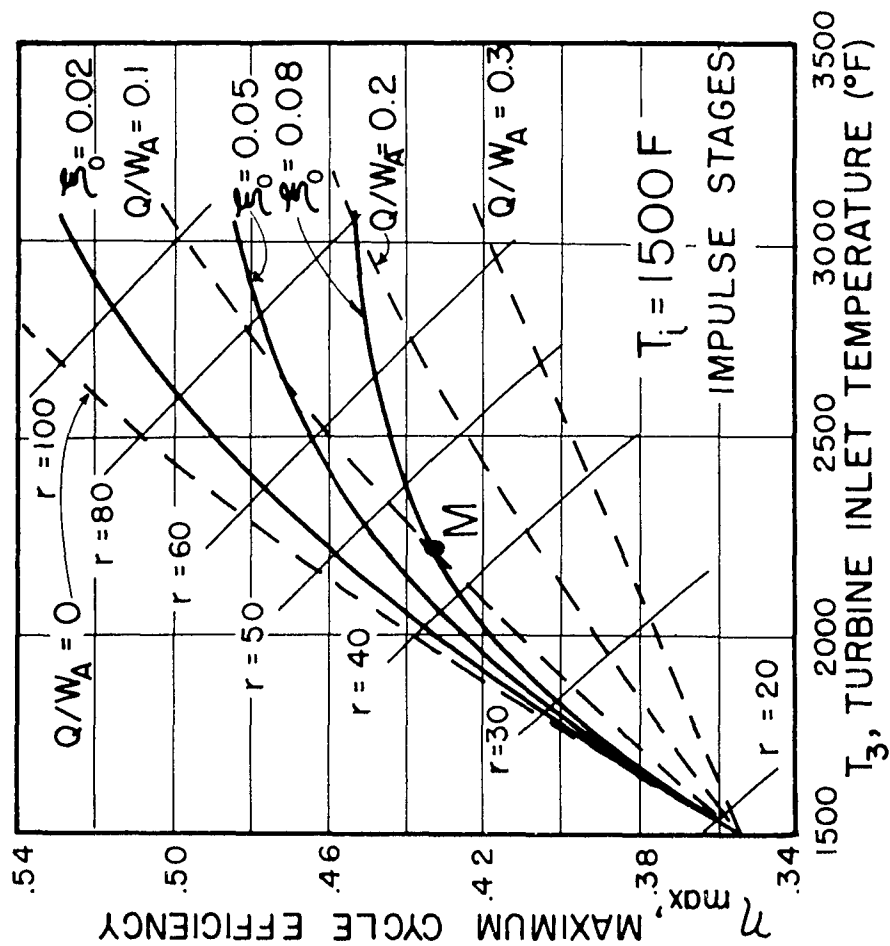


FIG. 3. MAXIMUM CYCLE EFFICIENCY VERSUS TURBINE INLET TEMPERATURE AT VARIOUS VALUES OF  $\zeta_0$ . IMPULSE STAGES WITH  $u = 1000$  ft./sec. AND  $T_i = 1500$  F.

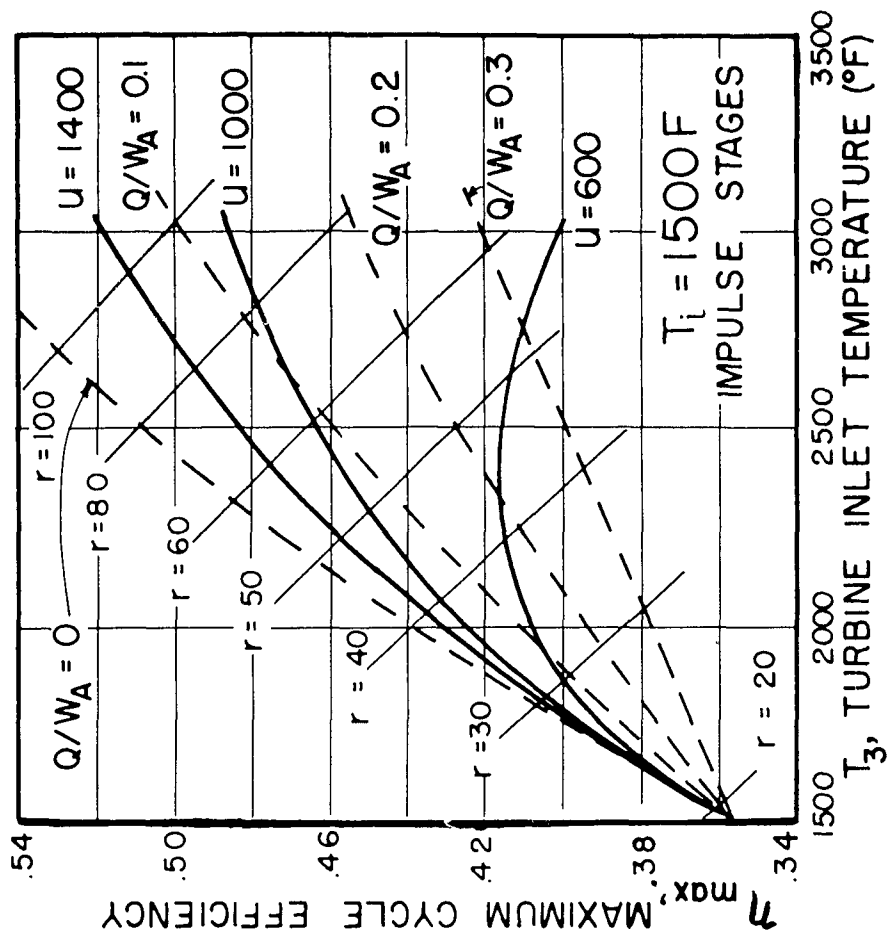


FIG. 4. MAXIMUM CYCLE EFFICIENCY VERSUS TURBINE INLET TEMPERATURE AT VARIOUS VALUES OF  $u$ . IMPULSE STAGES WITH  $\zeta_0 = 0.05$  AND  $T_i = 1500$  F.



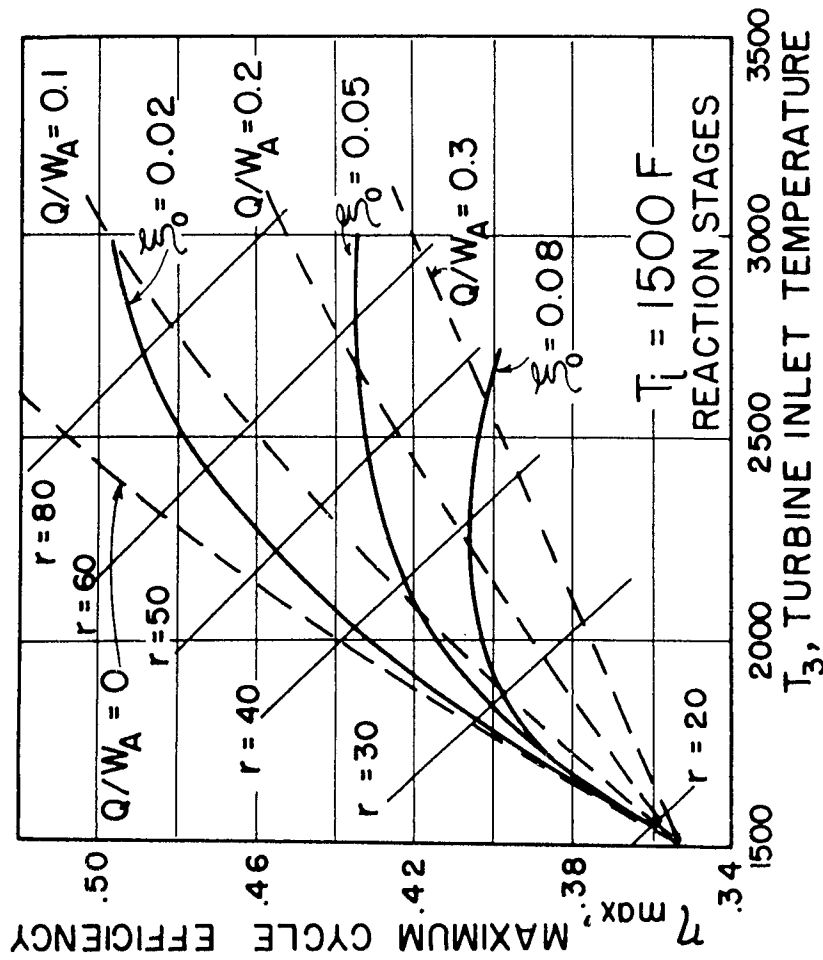


FIG. 5. MAXIMUM CYCLE EFFICIENCY VERSUS TURBINE INLET TEMPERATURE AT VARIOUS VALUES OF  $\xi_0$ . REACTION STAGES WITH  $u = 1000$  ft./sec. AND  $T_i = 1500$  F.

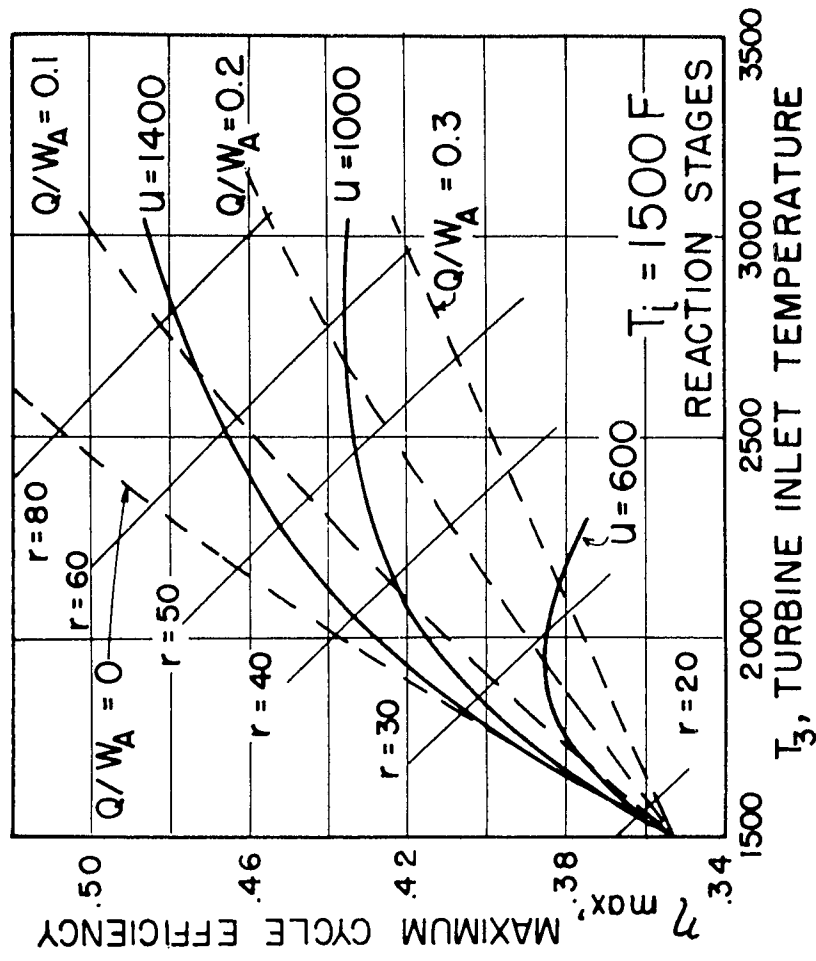


FIG. 6. MAXIMUM CYCLE EFFICIENCY VERSUS TURBINE INLET TEMPERATURE AT VARIOUS VALUES OF  $Q/W_A$ . REACTION STAGES WITH  $\xi_0 = 0.05$  AND  $T_i = 1500$  F.

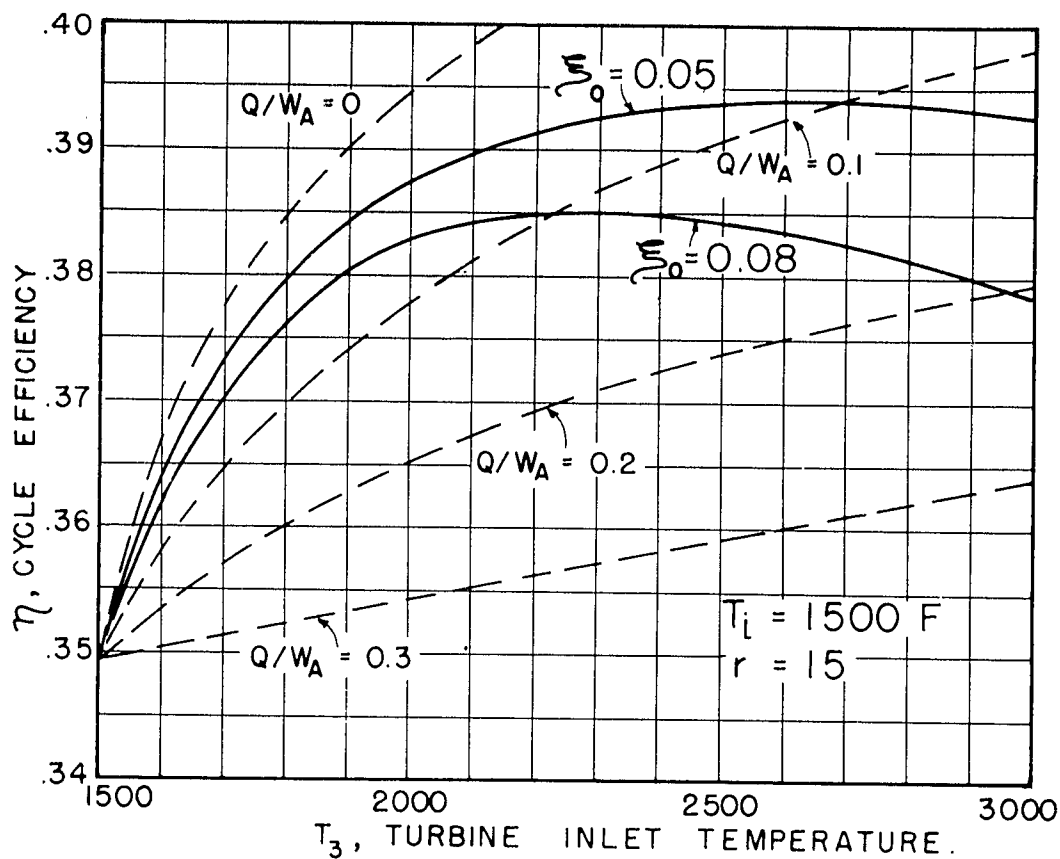


FIG. 7. CYCLE EFFICIENCY VERSUS  $T_3$  AT VARIOUS VALUES OF  $\xi_0$  WHEN  $r = 15$ . IMPULSE STAGES WITH  $u = 1000$  ft./sec. AND  $T_l = 1500$  F.

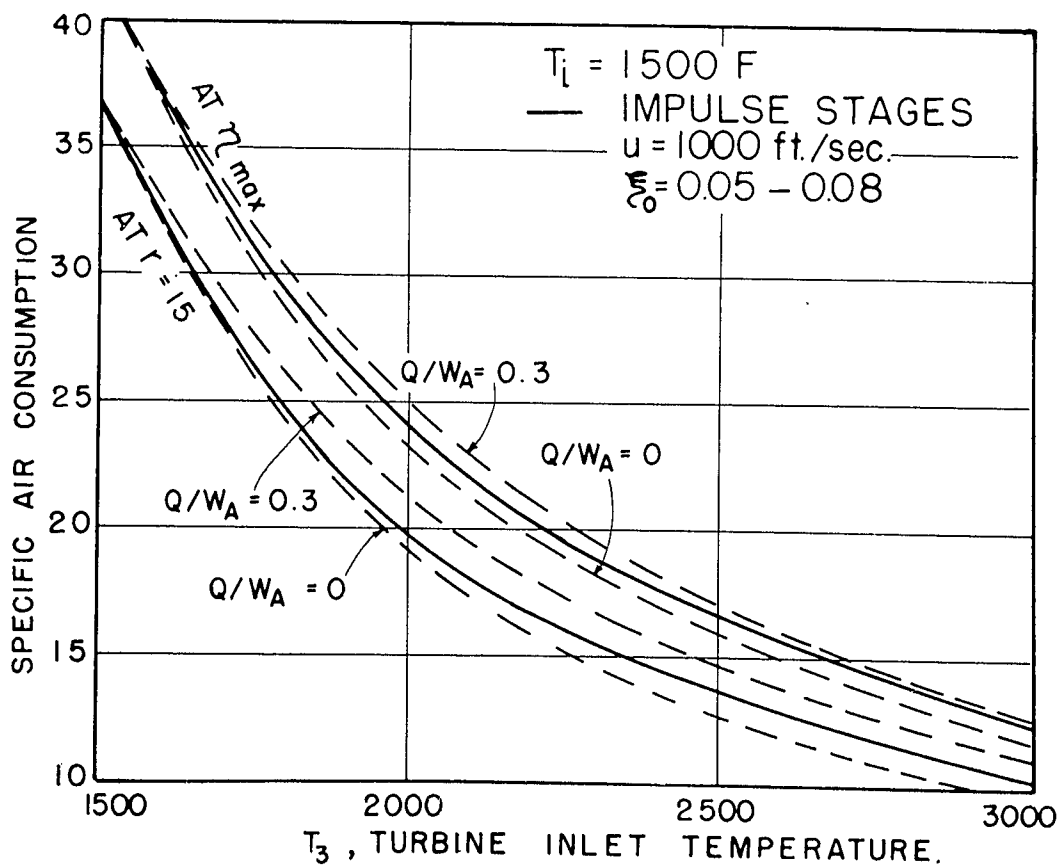


FIG. 8. SPECIFIC AIR CONSUMPTION VERSUS  $T_3$ . IMPULSE STAGES WITH  $u = 1000$  ft./sec.,  $\xi_0 = 0.05 - 0.08$  AND  $T_l = 1500$  F.

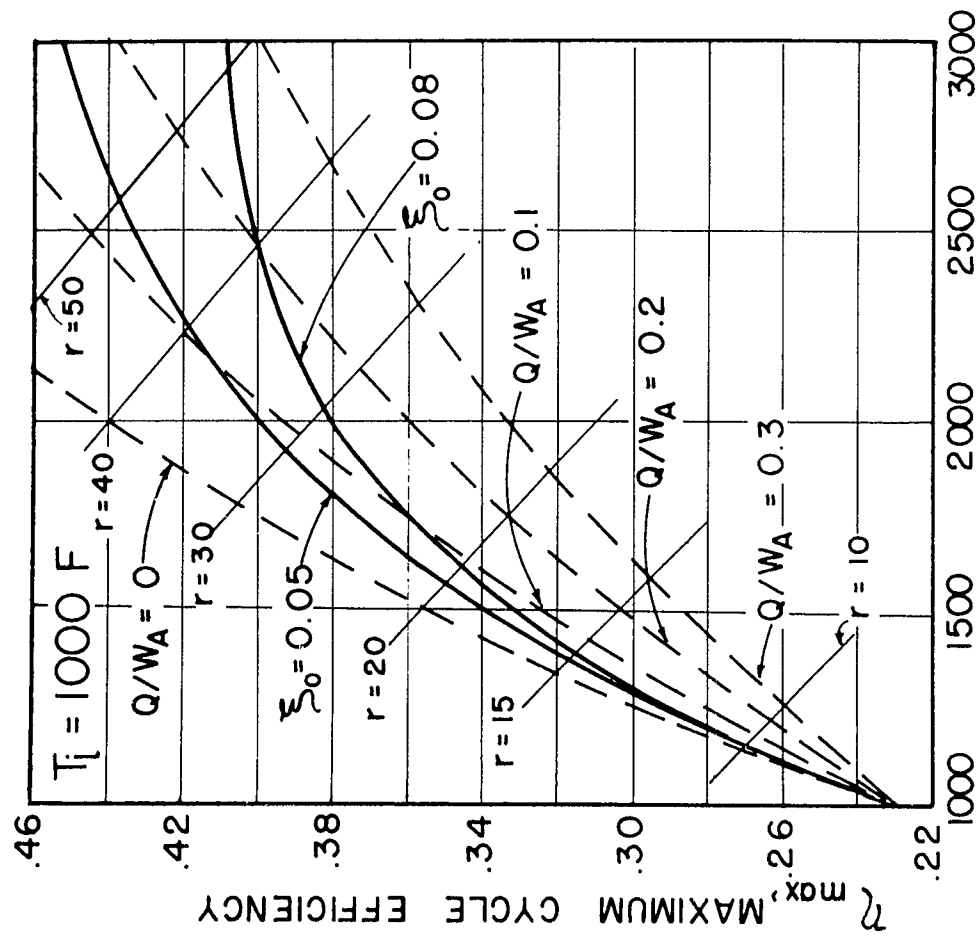


FIG. 9. MAXIMUM CYCLE EFFICIENCY VERSUS  $T_3$  AT VARIOUS VALUES OF  $\xi_0$ . IMPULSE STAGES WITH  $U = 1000$  ft./sec. AND  $T_i = 1000$  F

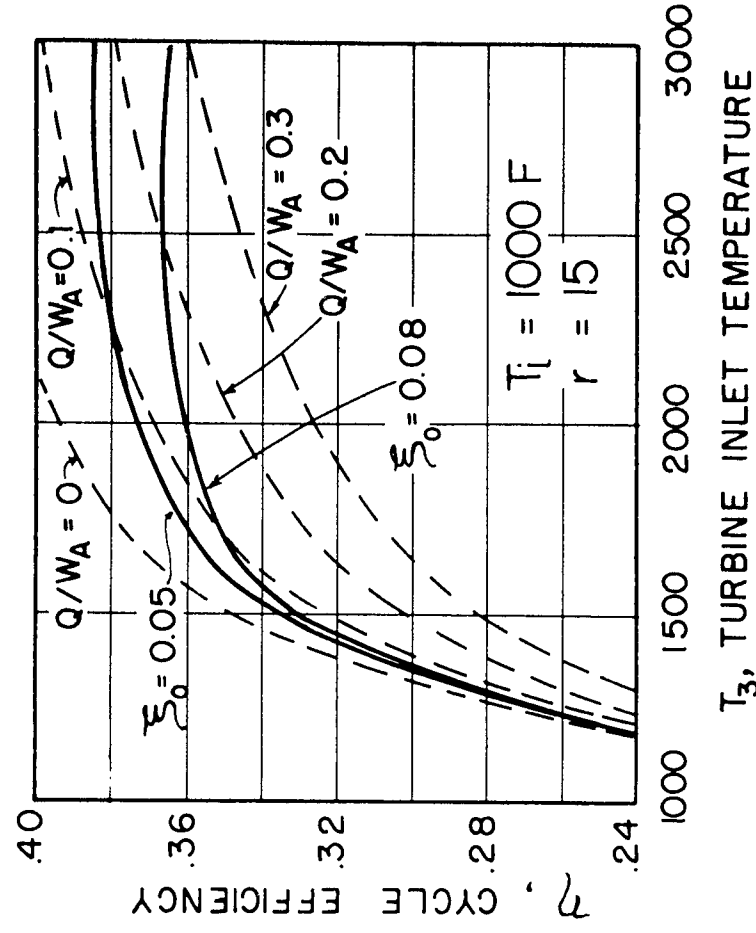


FIG. 10. CYCLE EFFICIENCY AT  $r=15$  VERSUS  $T_3$  AT VARIOUS VALUES OF  $\xi_0$ . IMPULSE STAGES WITH  $U = 1000$  ft./sec. AND  $T_i = 1000$  F.

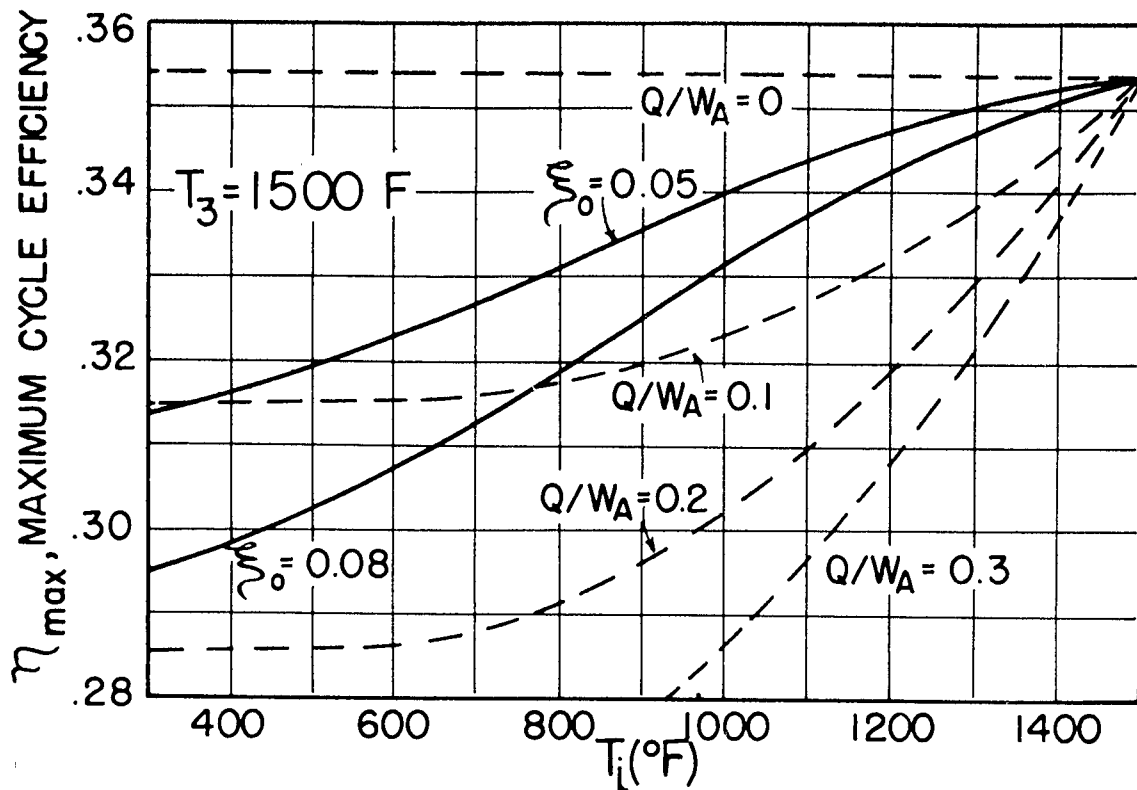


FIG. 11. MAXIMUM CYCLE EFFICIENCY VERSUS  $T_i$  AT VARIOUS VALUES OF  $\xi_0$ . IMPULSE STAGES WITH  $U = 1000$  ft./sec. AND  $T_3 = 1500$  F.

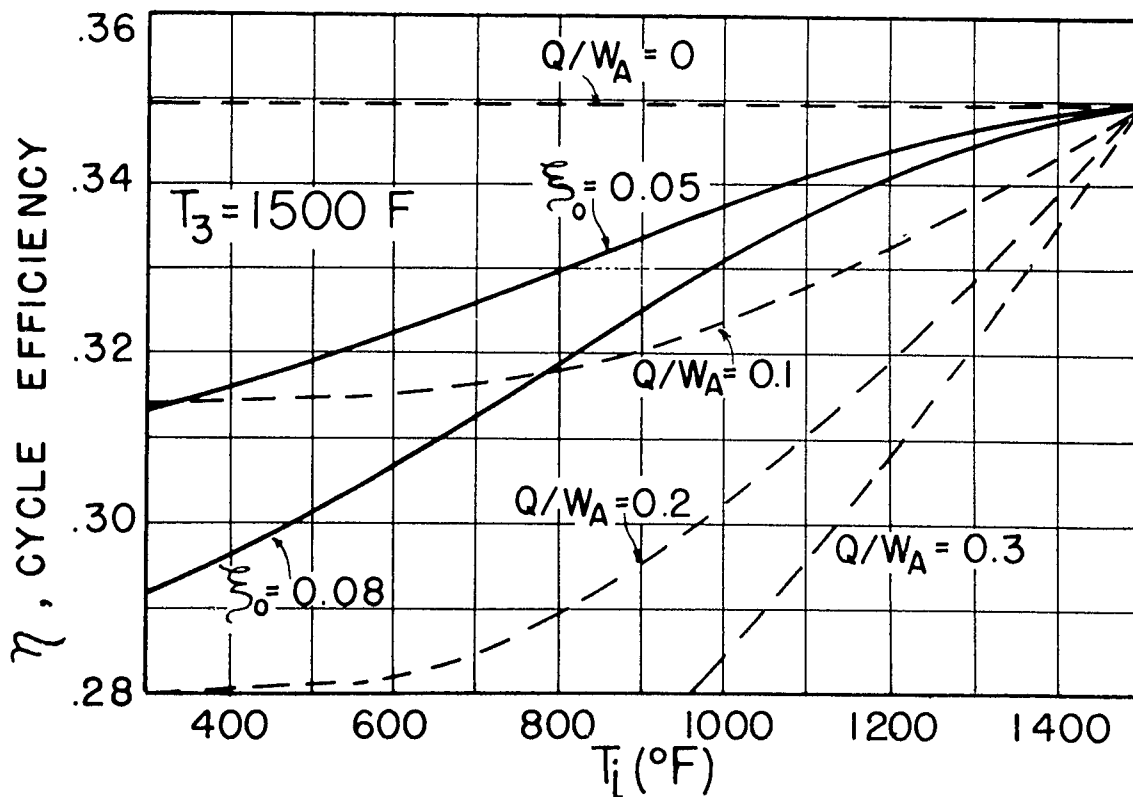


FIG. 12. CYCLE EFFICIENCY AT  $r=15$  VERSUS  $T_i$  AT VARIOUS VALUES OF  $\xi_0$ . IMPULSE STAGES WITH  $U = 1000$  ft./sec. AND  $T_3 = 1500$  F.

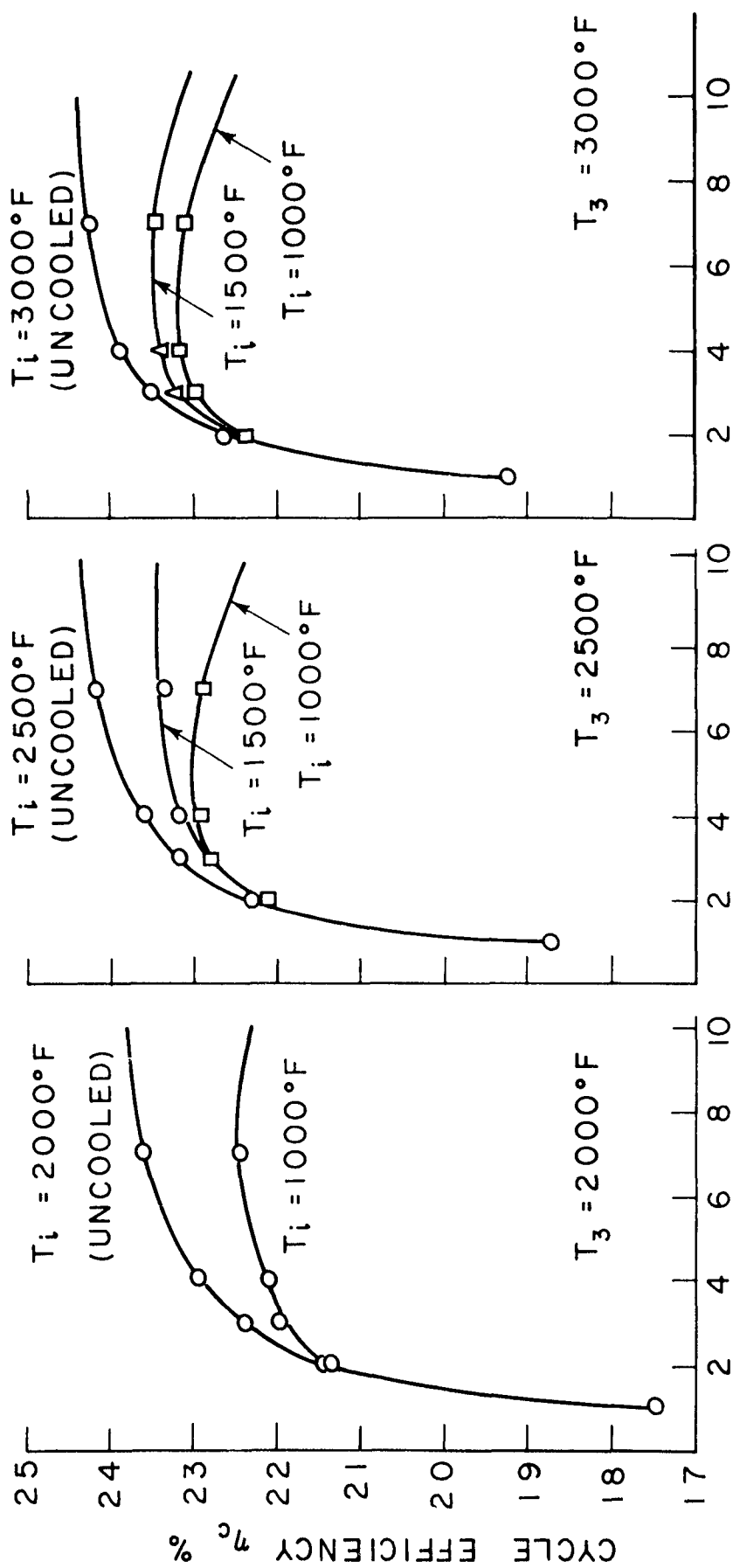


FIG. 13 - EFFECT OF FINITE NUMBER OF STAGES ON CYCLE EFFICIENCY FOR IMPULSE STAGES WITH  $\xi_0 = 0.08$ .

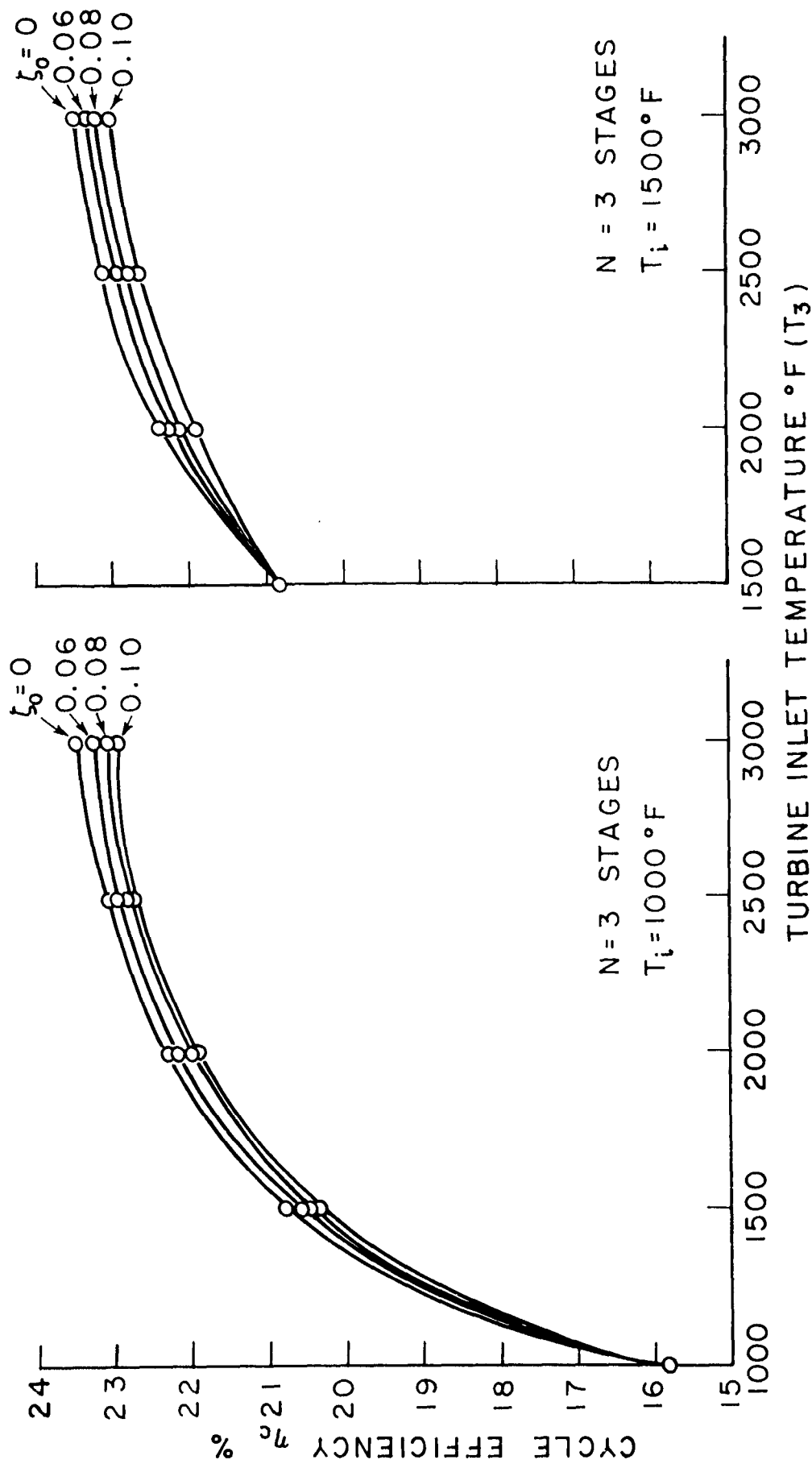


FIG. 14 - EFFECT OF  $T_3$  AND  $\xi_0$  ON EFFICIENCY OF A CYCLE WITH A TURBINE OF 3 IMPULSE STAGES AND PRESSURE RATIO OF 4.4.